ECE 383 - Embedded Computer Systems II
Lecture 23 - Direct Digital Synthesis

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Lesson Outline

- See schedule

- Lab 3

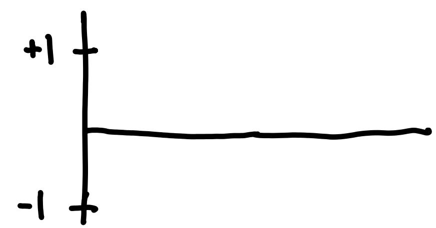
  - Write-up Due ~~COB~~ taps LSN ~~26~~ → 27 (today)
- Final Project Proposals Due BOC LSN ~~27~~ → 28
  - Revised proposals due BOC LSN ~~29~~ → 30
- HW12 Due BOC LSN~~27~~ 28 → Do during class today?

Today
- Fixed Point Arithmetic and Multiplication
- Direct Digital Synthesis
- Phase Increment → Really should be called frequency knob

Why DSS?

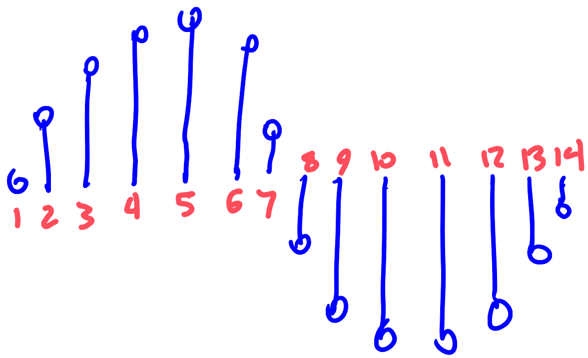
- **Lab#4: Function Generator** What is a function generator?
- **How to efficiently generate transcendental functions, like a sine wave?**
  - Useful in Final Project: Generate Sounds

- How to Generate Functions?
  - calculate in real-time?
  - approximate with CORDIC transform?
- LUT?
- What do all sine waves have in common?



- So just store ____ sine wave in a ____
  - playback at different ____
  - whoops, our F_s is ____ at ____ Hz → DSS

- Solution
  - jump through samples a ______________ (_____ increment)
  - for example,



suppose

$x=1 \rightarrow$ 220 Hz

$x=2 \rightarrow$ Hz

$x=4 \rightarrow$ Hz

$x=1.5 \rightarrow$ Hz

works until ________

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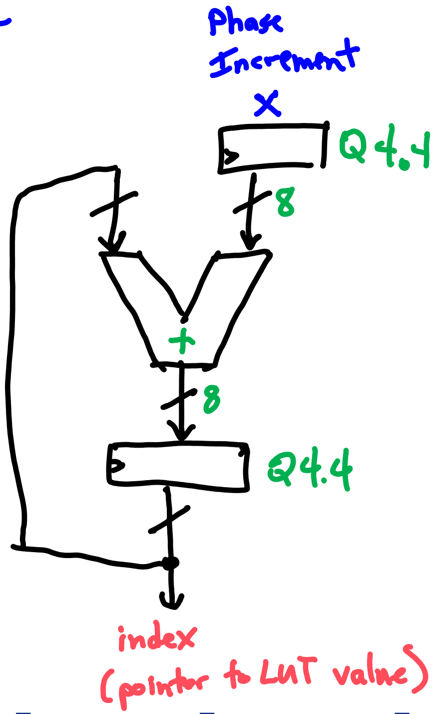
What is a radix point?
binary point?
decimal point?

After slide 23



Fixed Point

Register

1 0 0 1 0

what is this value?

Q format?

10010. Q_._

100.10 Q_._

.10010 Q_._

Fixed Point

- Binary Coded Number: 10010.
- How do you (formally) determine what decimal value it represents?
- You need the Equation:
  - Decimal value = sum(b_i*2^i)
- Where the sum ranges over all the bit positions i.

$$1*2^4 + 0*2^3 + 0*2^2 + 1*2^1 + 0*2^0 = 16 + 2 = 18$$

Fixed Point

- Well now generalize this idea to the right of the binary point and take a stab at what 1.11 means?
- Important to note is that to represent values less than 1 (to the right of the decimal point), negative indices need to be used

$$1*2^{0} + 1*2^{-1} + 1*2^{-2} = 1 + 0.5 + 0.25 = 1.75$$

- Q format:
  - 10110110 in Q4.4 is 1011.0110

go to slide 9

Fixed Point

- Lets now convert 1.53125 into binary.
- This is done by using the tried and true technique of finding the largest power of 2 that will fit into the number, subtracting it, and then continuing the conversion with the difference.
- This process stops when you get down to zero.
- To illustrate:
  - The largest power of two that fits into 1.53125 is $2^0 = 1.0$
  - The largest power of two that fits into 0.53125 is $2^{-1} = 0.5$
  - The largest power of two that fits into 0.03125 is $2^{-5} = 0.03125$
  - Thus the binary representation of 1.53125 is 1.10001

Q format X.X

Q format is a way to communicate where the binary point is in a binary number

Q4.4 10110110

Fixed Point

- As you can imagine, some rational real numbers do not have a rational binary representation.
- For example, the decimal number 0.1 cannot be represented as a finite binary string of 0's and 1's - it would repeat endlessly.
- You can give the conversion a try if you want to prove this yourself.

Fixed Point Arithmetic

Fixed Point Arithmetic

- Fixed point numbers vs floating point?

32-bit Register?

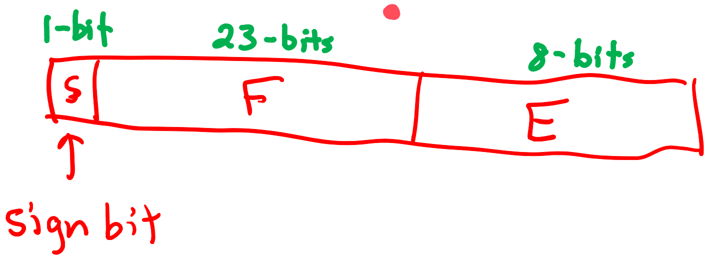
Q 32,0

Q 16,16

Q 0,32

Fixed:
-
-

Float:
-
-



$Value = (-1)^{S} \cdot 1.F \times 2^{E-128}$

$2^{255-128} = 2^{+127}$

$2^{0-128} = 2^{-128}$

- Can represent numbers with fractions even when hardware resources are limited or you would like to keep complexity to a minimum.

Fixed Point Arithmetic

Q format?

W7 W6 W5 W4 W3 W2 W1 W0 . F7 F6 F5 F4 F3 F2 F1 F0

- The 8 W-bits represent the whole portion
- The 8 F-bits represent the fractional portion
- The resulting 16-bit number can be manipulated as a whole with some minor book keeping to keep track of the binary point
- This is Q8.8 format

Fixed Point Arithmetic

- As an exercise determine the representation of:
  - 23.5 and 45.25 in Q8.8 format and then add them.

(23.5)

+ (45.25)

--

(68.75)

- What about the Binary Point? Does the hardware adder know where this is?

Fixed Point Arithmetic

- As an exercise determine the representation of:
  - 23.5 and 45.25 in Q8.8 format and then add them.

(23.5) 00010111.10000000

+ (45.25) 00101101.01000000

(68.75) 01000100.11000000

16 bits + 16 bit = ____ bits?

- What about the Binary Point? Does the hardware adder know where this is?

Fixed Point Multiplication

Fixed Point Multiplication

- Assume that 23 have a 4-bit representation where the binary point resides in the middle of the number.
- We will multiply 3.25 and 1.25. Use Q2.2

1101	(3.25)
x 0101	x (1.25)
---------	-------------

+

----

4 bits x 4 bits = _____ bits

Fixed Point Multiplication

- Assume that 23 have a 4-bit representation where the binary point resides in the middle of the number.
- We will multiply 3.25 and 1.25. Use Q2.2

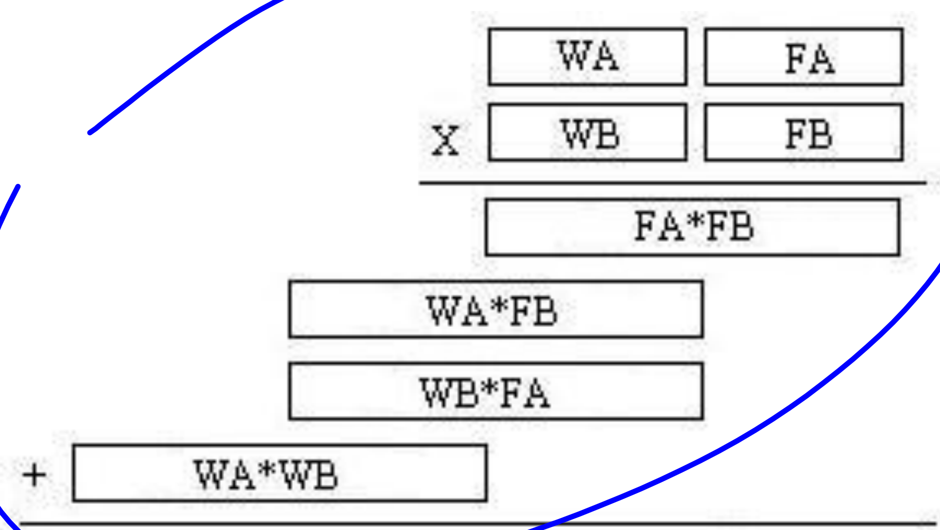
1101	(3.25)
x 0101	x (1.25)
---------	--------------
1101	1625
0000	6500
1101	+ 32500
+ 0000	--------------
-------------------- -----------	4.0625
100.0001	?

4 bits x 4 bits = ______ bits

Skip

Fixed Point Multiplication

- What if the multiplier is the wrong size?
- Lets consider the multiplication of two 16-bit fixed point numbers (representing angles) WA:FA and WB:FB.
- From our discussion above the product requires 32-bits to represent.



Direct Digital Synthesis

Direct Digital Synthesis

- Direct Digital Synthesis (DDS) is a technique to create periodic waveforms with very precise frequency control using a system with a fixed clock frequency.
- The periodic function is stored in a look-up table like the following for a sin wave.

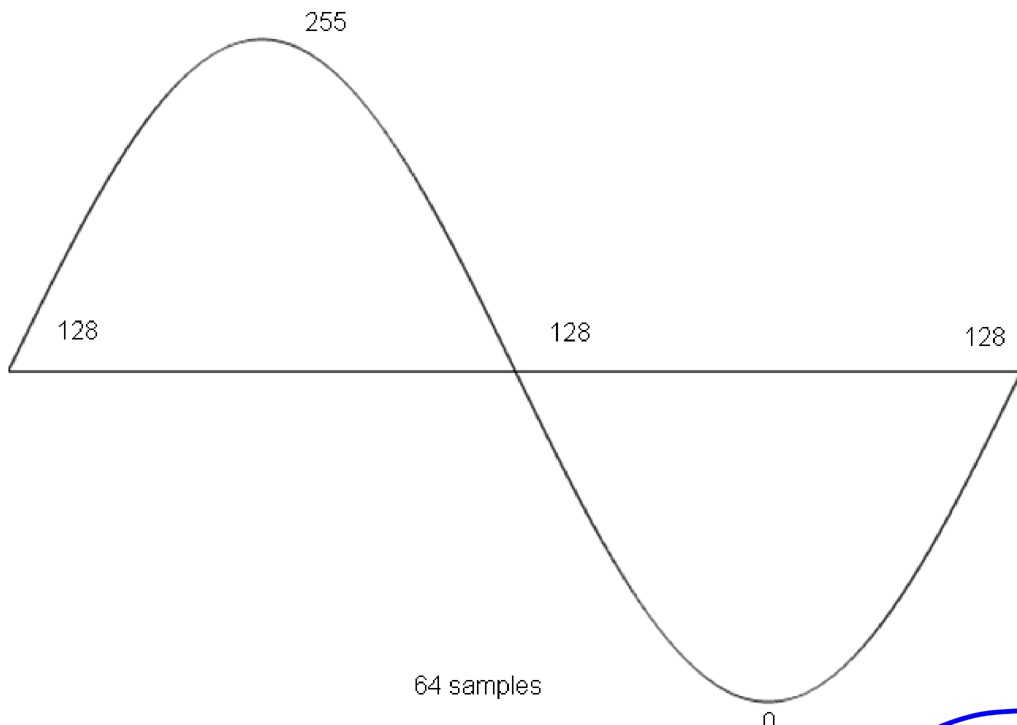
u8

int8 sin[64] = {128,141,153,165,177,189,200,210,219,227,235,241,246,250,253,255, 255,254,252,248,244,238,231,223,214,205,194,183,171,159,147,134, 122,109, 97, 85, 73, 62, 51, 42, 33, 25, 18, 12, 8, 4, 2, 1, 1, 3, 6, 10, 15, 21, 29, 37, 46, 56, 67, 79, 91,103,115,128};

Direct Digital Synthesis

One Cycle of our periodic wave

int8 sin[64] = {128,141,153,165,177,189,200,210,219,227,235,241,246,250,253,255, 255,254,252,248,244,238,231,223,214,205,194,183,171,159,147,134, 122,109, 97, 85, 73, 62, 51, 42, 33, 25, 18, 12, 8, 4, 2, 1, 1, 3, 6, 10, 15, 21, 29, 37, 46, 56, 67, 79, 91,103,115,128};



Our BRAM has room for 1024

2^{10}

- Table Length is a factor of 2^n (i.e. 2^6 = 64 samples).

$n = 6$

Example

Q6.0 ← only integers, no fraction

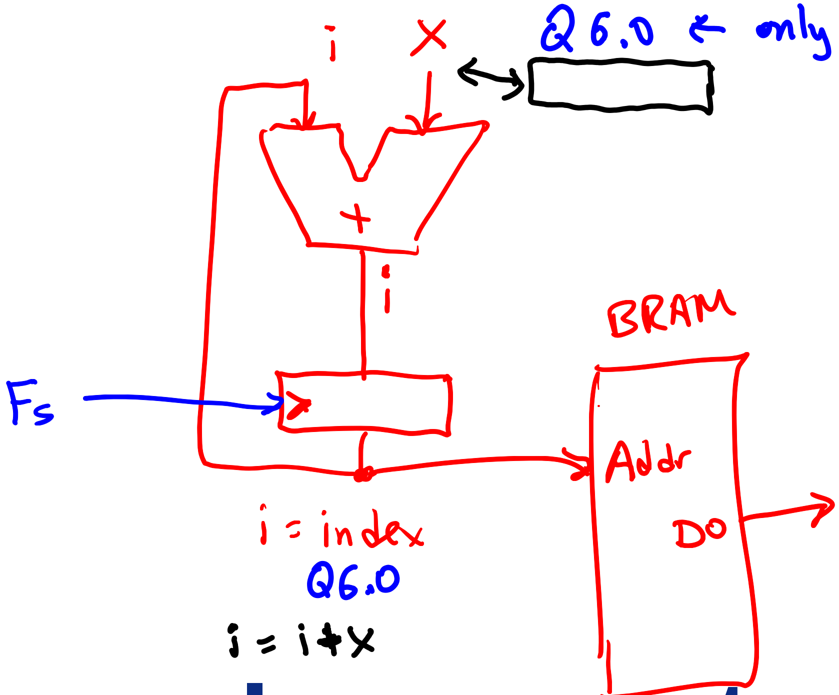
$x =$

$i_0 =$

$i_1 =$

$i_2 =$

$i_3 =$



Phase Increment

Examples

$x = 1$

$x = 2$

$x = 4$

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assume

$n = 6$

$2^n = 2^6 = 64$

Phase Increment = X

frequency Multiplier?

- Lets say that you could provide a new sample from the sin table at 48kHZ (through an interrupt) to the codec.

$T_s = \frac{1}{F_s} = \frac{1}{48kHz} = \_\_\_\_ \mu sec$

X=1.0

- If you incremented the pointer in the sin table by 1 on every interrupt. How long to get through the table? ← through one cycle

$\_\_\_\_ entries \cdot \_\_\_\_ \frac{\mu sec}{entry} = \_\_\_\_ mSec/cycle$

$frequency = \frac{1}{\_\_\_ mSec} = \_\_\_\_ Hz$

- If you incremented the pointer in the sin table by 2 every interrupt. How long to get through the table? X = 2.0

$\_\_\_\_ \cdot 20.833 \mu sec = \_\_\_\_ mSec/cycle$

$frequency = \frac{1}{\_\_\_} = \_\_\_\_ kHz$

Show spreadsheet DDS.xlsx

Phase Increment

- Lets say that you could provide a new sample from the sin table at 48kHZ (through an interrupt) to the codec.

Ts = 1/Fs = 1/48kHz = 20.8333 uS

- If you incremented the pointer in the sin table by 1 on every interrupt. How long to get through the table?

64*21uS = 1.3mS

$x = 1.0$

Generating a sine wave with a frequency of about 750Hz

- If you incremented the pointer in the sin table by 2 every interrupt. How long to get through the table?

32*21uS = 0.65mS

Generating one period of the sine wave for a frequency of about 1.5kHz. ← $x = 2.0$

Show spreadsheet

Phase Increment

- Using integer values for the increment we are limited to very coarse adjustments in the frequency.
- For example how could you use this schema to generate a sin wave with frequency of 1.0kHz?
- Well you would need to increment the pointer in the sin table by 1.5 every 21uS.
- And surprisingly, you can easily accomplish this using a fixed point representation.
- This fractional value is called the phase increment.

Phase Increment

- Lets look at how the phase increment, update rate, and size of the LUT are related to the output frequency.
- **1) Given a lookup table with 2^N values corresponding to one wavelength of a function.**
- **2) Given a sampling rate or a play back rate of f updates/second**
- **3) Given a phase increment x, which every 1/f is added to the index of the LUT.**

Magic Equation

$$\textbf{Freq} = \frac{f_s \textbf{ updates}}{\textbf{1 second}} * \frac{\textbf{x values}}{\textbf{update}} * \frac{\textbf{1 cycle}}{\textbf{2\^{}N values}} = \frac{f_s * x}{2^{\wedge}N} \textbf{ hz}$$

Example

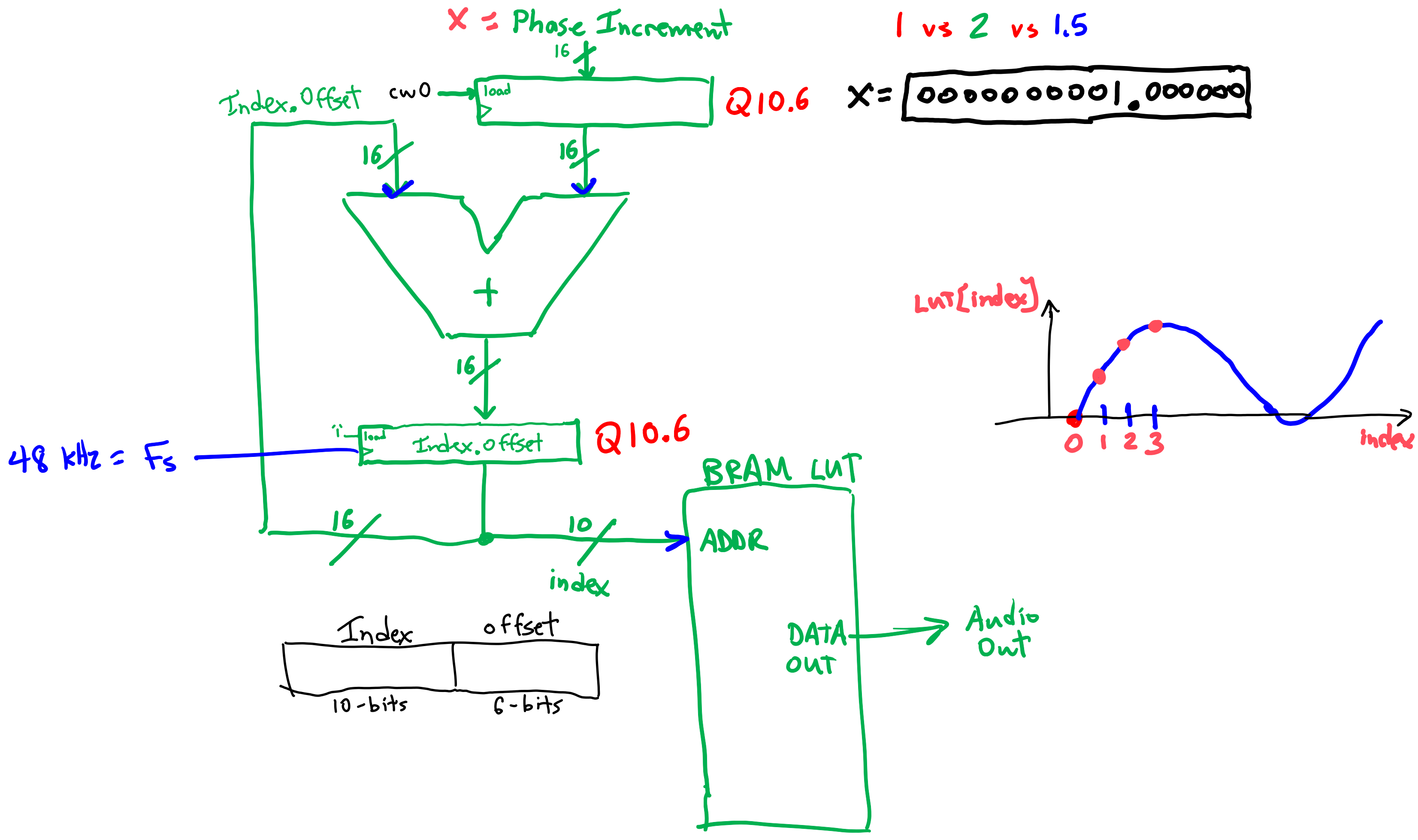
Draw the HW block diagram to implement DSS

Given: F_s = 48 KHz

2^N = 1024

Represent the phase increment X as a $\boxed{\text{Q10.6}}$ fixed point number?

Block Diagram



Given

$2^N = 1024$

Question

- **For this example, find x to generate 440 Hz as a Q10.6 fixed point number?**

$$\text{Freq} = \frac{f_s \text{ updates}}{1 \text{ second}} * \frac{x \text{ values}}{\text{update}} * \frac{1 \text{ cycle}}{2^\wedge N \text{ values}} = \frac{f^*x}{2^\wedge N} \text{ hz}$$

① 440 Hz = ——— ∴ x = ——— decimal .386

② In binary, Q10.6, x = .

③ What is this actually in decimal? x = .

④ What is the actual Frequency Produced?

f = ——— = . Hz

Questions

- Assuming an update rate of 48kHz, a LUT with 1024 entries, and a phase increment of x, expressed as a 10.6 fixed point number.
  - What is the maximum frequency we could generate? [assume Fs/4, not Fs/2]

$f_{max} = \frac{\quad}{\quad} = \quad kHz$

  - What is the phase increment X for this max freq?

$12k = \frac{\quad}{\quad} \quad X =$

Questions

- Assuming an update rate of 48kHz, a LUT with 1024 entries, and a phase increment of x, expressed as a 10.6 fixed point number.
  - What is the minimum frequency we can generate?

$x = \boxed{\quad\quad\quad\quad . \quad\quad}$

$f_{min} = \frac{.}{\quad\quad} = \quad . \quad$ Hz

Questions

- Assuming an update rate of 48kHz, a LUT with 1024 entries, and a phase increment of x, expressed as a 10.6 fixed point number.
  - What is the smallest change in frequency we can make with the phase increment?

x = 0.000001 .73 Hz

0.000010

Questions

- Assuming an update rate of 48kHz, a LUT with 1024 entries, and a phase increment of x, expressed as a 10.6 fixed point number.
  - What is the frequency produced when we make the phase increment X = 1.0?

$f = \frac{\quad}{\quad} = \quad \text{Hz}$

Questions

- Assuming an update rate of 48kHz, a LUT with 1024 entries, and a phase increment of x, expressed as a 10.6 fixed point number.
  - How did I arrive at the format of the phase increment? Q10.6?

10 → ?

.6 → ?

10+6 → ?

$$f = \frac{F_s \cdot x}{2^N}$$

Final Project Proposal

- **Due BOC next lesson**

and HW12